On the Estimation of the Domain of Stability for Nonlinear Systems

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Abstract—**This paper deals with the attraction domain estimation of the stable equilibrium point of nonlinear systems. An algebraic approach is firstly studied to consider a sufficient domain characterized by a ball centred in the origin equilibrium stable point and of a suitable calculated radius R_0. The second proposed approach is based on the resolution of a convex optimization problem to maximize the radius of a sufficient obtained domain of attraction. The third method consists on computing a quadratic Lyapunov function which also optimizes the volume of the estimated domain of attraction. A comparative study is developed to illustrate the benefits of the studied methods so as to obtain the largest domain of attraction.**

Keywords−nonlinear system, asymptotic stability, domain of attraction, Lyapunov Function, Polynomial system and LMI optimisation

I. INTRODUCTION

The estimation of domain of attraction (DA) of equilibrium point is one of the most important problem when considering the nonlinear systems analysis because of the fact that in many applications the stability properties of a nonlinear system do not remain globally. In recent years, intensive researches have been made treating this important problem([5],[7],[10],[11]). Many methods have been proposed in the literature without obtaining an exact DA of the equilibrium stable point[20]. As a consequence, the estimation of the DA is still an open problem for the reason that is difficult and impractical to determine the exact DA . The research has been made in order to enlarge the DA without needs to reaching the exact one. In this scenario, the most important methods are based on searching some Lyapunov Functions (LF) structures able to get a maximized DA ([19],[20]. These methods are known as 'Lyapunov methods'([1],[9]). Unfortunately, there is no systematic algorithm of considering suitable LF and also it's not evident to get an algebraic relation between the LF and nonlinearities of the system. Recently, the majority of works are based on the nonlinear decomposition approaches leading generally to an application of LMI-based conditions ([6],[14]). A part of these methods uses sum of squares (SOS) decomposition of the Lyapunov stability criterion which are then programmed ([15],[16],[17],[21]). The second part of these approaches employs a decomposition of the nonlinear system model in order to get an LMI formulation of the Lyapunov stability conditions. In this paper, we attempt to make a point about these different original methods for estimating the DA of nonlinear systems. Two of them are based on the definition of a quadratic LF and the third is classed as a 'nonLyapunov' method applied particularly for polynomial models. For instance, a computational of the quadratic LF which maximizes the volume of the estimate of DA proposed by Chesi [4] uses a semi convex approach based on Linear Matrix Inequality (LMI). A double non-convex optimisation is considered to solve the problem. Moreover, Chesi proposes a relaxed criterion for obtaining an effective starting candidate of the optimal quadratic Lyapunov function in the case of odd polynomial system by considering the same class of polynomial systems. An interesting analytic method based on the Gronwell-Bellman Lemma allows the guaranteed DA of nonlinear systems is developed in [3]. One of the recent methods has been studied in [13] consists on the optimization of the quadratic Lyapunov function which enlarges the estimate domain of attraction for the particular class of polynomial nonlinear systems. The enlargement and the exactness of the asymptotic stability region will be considered as the main qualitative criterion of the comparison of the studied methods when applied on predator-prey nonlinear system. A qualitative comparison study is finally proposed in order to conclude about the efficiency and the feasibility of each method.

II. DESCRIPTION OF THE STUDIED POLYNOMIAL SYSTEM

Let consider the polynomial systems described by:

$$\dot{X} = f(X) \tag{1}$$

Where f is a polynomial function of X.

$$f(X) = \sum_{i=1}^{r} A_i X^{[i]} = \sum_{i=1}^{r} \tilde{A}_i \tilde{X}^{[i]} \tag{2}$$

with:
$X = [x_1 \ldots x_n]^T \in IR^n$
$A_{i=1,\ldots,r} \in IR^{n \times n^i} (resp. \tilde{A}_{i=1,\ldots,r} \in IR^{n \times n_i})$ are constant matrix.
$X^{[i]}_{i=1,\ldots,r} \in n^i$ are the ith order Kronecker power of the state X described by:

$$\begin{cases} X^{[0]} = 1 \\ X^{[j]} = X^{[i-1]} \otimes X = X \otimes X^{[i-1]}; i \geq 1 \end{cases} \quad (3)$$

Where $\otimes$ design the kronecker product. $\tilde{X}^{[i]}_{i=1,\ldots,r} \in {}^{n_i}, n_i = \begin{pmatrix} n+i-1 \\ i \end{pmatrix}$ is the non-redundant power of the vector X defined by:

$$\tilde{X}^{[1]} = X^{[1]} = X$$

$$\begin{aligned} \forall i \geq 2 \quad \tilde{X}^{[i]} &= [x_1^i, x_1^{i-1}x_2, \ldots, x_1^{i-1}x_n, x_1^{i-2}x_2^2 \\ &\ldots, x_1^{i-2}x_2x_3, \ldots, x_1^{i-2}x_n \\ &\ldots, x_1^{i-2}x_n^2, \ldots, x_1^{i-3}x_2^3, \ldots, x_n^i]^T \end{aligned} \quad (4)$$

Relation between redundant power and non-redundant power for the state vector can be defined by:

$$\begin{aligned} \forall i \in \mathrm{N}, \exists\, T_i \in IR^{n^i \times n_i} \\ X^{[i]} = T_i \tilde{X}^{[i]} \end{aligned} \quad (5)$$

A method of construction of T_i is developed in [2].

III. METHODS OF ESTIMATION OF THE ATTRACTION DOMAIN

A. first approach

Consider the system described by the following polynomial representation:

$$\dot{X} = \sum_{i=1}^{r} F_i \tilde{X}^{[i]} \quad (6)$$

The stability domain of the stable equilibrium point of the system (6), where the linear part is asymptotically stable, is given by the following theorem [3]:

Theorem 1: Consider the system defined by (5) where $F1$ is Hurwitz matrix that verifies lemma1 (appendix). Such system is exponentially stable in $B(O, R_0) = \{X \in IR^n : \|X\| < R_0\}$ with R_0 the unique positive solution of the following equation:

$$\sum_{k=2}^{r} \|F_k\| c^{k-1} R_0^{k-1} - \frac{\alpha}{c} = 0 \quad (7)$$

❏

B. Second approach

Consider the system described by:

$$\dot{X} = \sum_{i=1}^{r} \tilde{A}_i \tilde{X}^{[i]} = A_1 X + \sum_{i=2}^{r} \tilde{A}_i \tilde{X}^{[i]}, X \in IR^n \quad (8)$$

Suppose that the linear part of the system (8) is asymptotically stable, that A_1 is a Hurwitz matrix. We define the asymptotic stability domain D with the following invariant set:

$$D = \left\{ X \in IR^n, V(X(t, t_0, X_0)) < c, \dot{V}(X) < 0 \right\} \quad (9)$$

The stability domain is formed by a sphere centered at the origin and radius R_0, denoted:

$$D = \mathrm{B}(O, R_0) = \{X_0 \in IR^n; \|X_0\| < R_0\} \quad (10)$$

The strategy consist on determining the largest radius R_0 by using the quadratic Lyapunov function:

$$V(X) = X^T P X \quad (11)$$

where P is $(n \times n)$ positive definite matrix. Asymptotic stability of the equilibrium of the system (8) is guaranteed when the derivative $\dot{V}(X)$ of $V(X)$ is negative definite:

$$\begin{aligned} \dot{V}(X) &= X^T P \dot{X} + \dot{X}^T P X \\ &= X^T \left(P A_1 + A_1^T P \right) X + 2 \sum_{i=2}^{r} X^T P \tilde{A}_i \tilde{X}^{[i]} \end{aligned}$$

The following theorem resumes the approach of determination of stability domain:

Theorem 2: Consider the system described by:

$$\dot{X} = \sum_{i=2}^{r} \tilde{A}_i \tilde{X}^{[i]}, X \in IR^n \quad (12)$$

where A_1 is a Hurwitz matrix. The system is asymptotically stable in a ball $B(O, R_0)$ with R_0 is the unique positive solution of:

$$\sum_{i=2}^{r} \|\mathrm{A}_i(P, \tilde{\mu}_{i+1})\| \left(\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \right)^{i-1} R_0^{i-1} - \lambda_{\min}(Q) = 0 \quad (13)$$

with:
$$\mathrm{A}_i(P, \tilde{\mu}_{i+1}) = 2vec^T \left(P \tilde{A}_i T_i^+ \right) + \tilde{\mu}_{i+1}^T D_{i+1}^T$$
and

$$PA + A^T P = -Q$$

❏

A method of calculating $\tilde{\mu}_{i+1}$ and D_{i+1} is developed in [13].

The strategy consists on the determining of a maxima stability radius R_0^* by maximization of the solution of (13) based on variables P and $\tilde{\mu}_{i+1, i=2,\ldots r}$

This approach consists to divide (13) by $\lambda_{\max}(P)$ as following:

$$\sum_{i=2}^{r} \frac{\|\mathrm{A}_i(P, \tilde{\mu}_{i+1})\|}{\lambda_{\max}(P)} \left(\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \right)^{i-1} R_0^{i-1} - \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} = 0 \quad (14)$$

This equation has a unique positive solution which is equivalent to the solution of the equation (13).

The problem is to minimize $\frac{\|\mathrm{A}_i(P, \tilde{\mu}_{i+1})\|}{\lambda_{\max}(P)} \left(\sqrt{\frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}} \right)^{i-1}$ and maximize $\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$.

$\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$ is maximum for $Q = I$ which leads to determine R_0^* the unique positive solution of:

$$\sum_{i=2}^{r} \alpha_i^* R^{i-1} - 1 = 0 \quad (15)$$

where

$$\alpha_i^* = \min_{\tilde{\mu}_{i+1}} \left[\|A_i(P^*, \tilde{\mu}_{i+1})\| \left(\sqrt{\frac{\lambda_{\max}(P^*)}{\lambda_{\min}(P^*)}} \right)^{i-1} \right] \quad (16)$$

P^* is the solution of :

$$-I = P^* A_1 + A_1^T P^* \quad (17)$$

C. Third approach

This technique allows to compute an Optimal Quadratic Lyapunov Function (OQLF) for polynomial systems. For this purpose the author uses in [4] a Linear Matrix Inequalities (LMI) approach dealing with non-convex distance problems. The optimal estimate of the DA for fixed LF is computed avoiding local minima via a one-parameter sequence of LMI's requiring a low computational burden.Otherwise, an expanded criterion for odd polynomial systems is considered in order to get a good starting point for the non-convex step. The developed criterion is based on maximising the volume of the region where time derivative of the LF is negative. We consider the polynomial system defined as:

$$\dot{X} = AX + \sum_{i=2}^{m_f} f_i(X) \quad (18)$$

where $X \in IR^n$, $f_i(X)$ is an homogeneous vectors of degree i, the origin is supposed locally asymptotically stable equilibrium point. The quadratic Lyapunov function $V(X, P) = X^T P X$ is considered, P is considered positive defined and chosen such us the time derivative quadratic function expressed by:

$$\dot{V}(P, X) = 2X^T P \left[AX + \sum_{i=2}^{m_f} f_i(X) \right] \quad (19)$$

is locally negative definite The set of all P verifying precious hypothesis can be characterized as:

$$P = \left\{ P = P^T \in IR^{n \times n} : PA + A^T P = -Q, Q > 0 \right\} \quad (20)$$

By considering the double sets induced by $\nu(P, c)$:

$$\nu(P, c) = \{ X \in IR^n; V(X, P) \le c \} \quad (21)$$

and

$$D(P) = \left\{ X \in IR^n; \dot{V}(X, P) \le 0 \right\} \cup \{0\} \quad (22)$$

then $\nu(P, c)$ is concluded as an estimate of domain of attraction of the origin once $\nu(P, c) \subseteq D(P)$. The computation of the OQLF is resumed in solving a double non-convex optimisation problem. the first requires the solution of a non-convex distance problem resumed by:

$$\begin{cases} \gamma(P) = \inf_{X \in IR^n \setminus \{0_n\}} X^T P X \\ \dot{V}(P, X) = 0 \end{cases} \quad (23)$$

The second is computed in order to maximize the volume of the DA and can be started by:

$$\begin{aligned} P^* &= \arg\max_{P \in P} \delta(P) \\ \delta(P) &= \sqrt{\frac{(\gamma(P))^n}{\det(P)}} \end{aligned} \quad (24)$$

$\delta(P)$ is the volume of the set $\nu(P; \gamma(P))$ up to a scale factor depending on the state dimension n.

IV. NUMERICAL EXAMPLE

In the sequel, we investigate the efficiency of the presented approaches by studying a benchmark of biological process known as predator prey system. The aim of this paragraph is to make a framework of a comparative study allowing the evaluation of the different approaches. Our main criteria for performance is the size of the estimated DA.
Let us consider the following continuous-time system:

$$\begin{cases} \dot{x}_1 = -3x_1 + 4x_1^2 - 0.5x_1x_2 - x_1^3 \\ \dot{x}_2 = -2.1x_2 + x_1x_2 \end{cases} \quad (25)$$

The aforementioned system represents the predator prey model as mentioned in [8], where the DA shape is well defined (See [8]). It is readily to check the following assertion:
The origin $(0, 0)$ is asymptotically stable. The point $(2.1, 1.98)$ as a point of interest. The point $(1, 0)$ and the point $(3, 0)$ are unstable. The Local DA we are looking for is a region around the asymptotic stable equilibrium point characterized by the following the coordinates $(2.1, 1.98)$. Let us consider the following variable change:

$$\begin{cases} X_1 = x_1 - 2.1 \\ X_2 = x_2 - 1.98 \end{cases} \quad (26)$$

As a result we obtain the deviation model:

$$\dot{X} = AX + \tilde{A}_2 \tilde{X}^{[2]} + \tilde{A}_3 \tilde{X}^{[3]} \quad (27)$$

where:
$$A = \begin{bmatrix} -3 & 0 \\ 0 & -2.1 \end{bmatrix} \tilde{A}_2 = \begin{bmatrix} 4 & -0.5 & 0 \\ 0 & 1 & 0 \end{bmatrix} \text{ and }$$
$$\tilde{A}_3 = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

1) First approach: Based on Theorem (1),the DA can be defined by:

$$\Gamma = \{ X \in {}^n; \|X\| \le R_0 \} \quad (28)$$

where R_0 is the unique positive solution of the equation (6).Let us remember that as stated in the Grownwell-bellman lemma the constants α and c are expected to satisfy the following inequality: $\exp(\alpha t) \|\exp(At)\| \le c$ In practice α and c are determined regarding the following conditions:
-The function $y = \exp(\alpha t) \|\exp(At)\|$ has to converge to a fixed value.

-The ratio $\frac{\alpha}{c}$ should be the greatest as possible.

Based on the numerical simulation we found the following values :

$\alpha = 2$ and $c = 1$

Thus the estimated DA is a circle where center the origin and the radius is $R_0 = 0.44$.

Figure 1 shows the DA obtained via the first approach procedure.

2) Second approach: By considering the polynomial representation (27) the second leads to conclude that the origin is asymptotically stable for all initial states belonging to the ball of center the origin and of radius $R_0^* = 0.52$. The optimized parameters related to the implementation of this approach are given by:

$$P^* = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.25 \end{bmatrix}$$

$\tilde{\mu}_3 = [0.0018; 0.0081; -0.0015; 0.0037]^T$

$\tilde{\mu}_4 = [0; 0.0031; 0.0012; 0.0012; 0.0002; -0.0019; 0.0004 ; -0.006; 0.0003; 0.0026; 0.062]^T$

Figure 2 shows the DA obtained by applying the second approach procedure.

3) Third approach: We consider the polynomial predator prey system defined by:

$$\dot{x} = Ax + f_2(x) + f_3(x) \tag{29}$$

The origin is locally asymptotically stable equilibrium point and $f_2(x)$ and $f_3(x)$ represent nonlinear vectors respectively of degree 2 and 3.

$f_2(x) = [4x_1^2 - 0.5x_1x_2 \ \ x_1x_2]^T$

$f_3(x) = [-x_1^3 \ 0]^T$

We consider the quadratic function $V(P,X) = X^T PX$, where $P > 0$ and its time-derivative:

$$\dot{V}(P,X) = 2X^T P[AX + f_2(x) + f_3(x)] \tag{30}$$

is locally negative definite. The computation of $\gamma(P)$ presented by (23) is performed by initializing $c = 0.01$. The optimal quadratic Lyapunov function which minimizes the volume of the DA (24) is initialized by choosing $P = \begin{bmatrix} 0.16 & 0 \\ 0 & 0.25 \end{bmatrix}$

Figure (3) shows the result obtained by the third approach for the considered system.
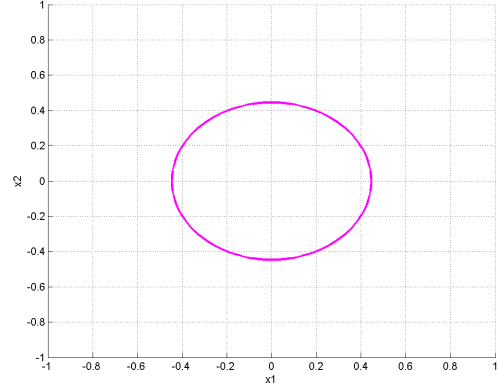

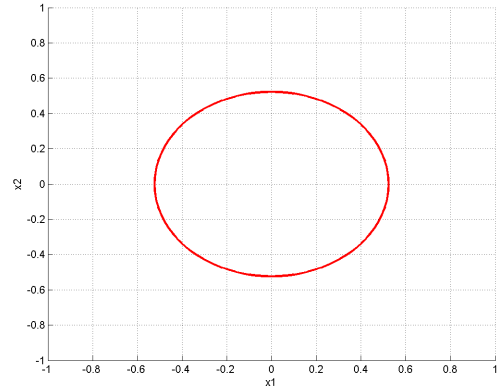Fig. 1.   First approach domain of attraction of predator prey system

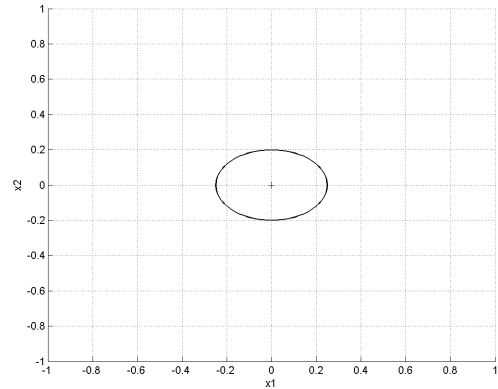Fig. 2.   Second approach domain of attraction of predator prey system

Fig. 3.   Third approach domain of attraction of predator prey system

V. RESULTS DISCUSSION

This paragraph is dedicated to comment the studied approaches regarding the results obtained by leading a numerical simulation. It is obvious that the largest domain is estimated when implementing the second approach. However, the first

approach shows the advantage of being simple and straightforward in its application. Indeed, this approach doesnt suggest any constraints to initialize the starting parameters. The most delicate step to exploit the second approach can be considered as a fact of defining arbitrary vectors. Such vectors have crucial importance in converging to an optimal solution. Equally important, the major difficulty in applying the third approach consists in performing a double convex optimization problem.

VI. CONCLUSION

In this work, methods for estimating DA of nonlinear polynomial systems have been presented. The first approach based on the Gronwell-Bellman lemma deals with a DA of circular form around the stable equilibrium point. The second approach yields a simple and sufficient DA for nonlinear continuous systems.The radius of stability determined by this approach has been maximized by solving a convex optimisation problem. A computation of the volume of the estimate DA has been studied in the third part of this paper in order to solve a double non-convex optimization problem based on LMI. To show the efficiency of the studied methods, a numerical study has been considered on the nonlinear polynomial predator prey system. To sum up, the second approach may lead to better and more efficient results if a good choice of estimated parameters is made.

APPENDIX

Lemma 1: Consider the nonlinear system defined by:

$$\dot{X} = F_1 X + g\left(t, X\right), X \in IR^n \tag{31}$$

Suppose that the linear part of the system (31) is asymptotically stable, that $F1$ is Hurwitz and the nonlinear part $g\left(X, t\right)$ verifies the following inequality:

$$\left\| g\left(t, X\right) \right\| \leq b \left\| X \right\|, b \in_+^* \tag{32}$$

Let $\Phi\left(t, t_0\right)$ the transition matrix of the linear part :

$$\Phi\left(t, t_0\right) = \exp\left(F_1\left(t - t_0\right)\right) \tag{33}$$

Let c and α two positive real numbers satisfying:

$$\Phi\left(t, t_0\right) \leq c \exp\left(-\alpha\left(t - t_0\right)\right) \forall t \geq t_0 \tag{34}$$

The solution $X\left(t\right)$ of (31) verifies the following inequality:

$$\left\| X\left(t\right) \right\| \leq c \left\| X\left(t_0\right) \right\| \exp\left(\left(cb - \alpha\right)\left(t - t_0\right)\right) \tag{35}$$

Then if $b < \frac{\alpha}{c}$,the system (31) is exponentially stable.

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